# ABSTRACT

of the dissertation submitted for the degree of Doctor of Philosophy (PhD) in the educational program 8D07101 – Automation and Control

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## APPLICATION OF NONLINEAR METHODS TO CONTROL A MULTI-LINK ROBOTIC ARM

**The main idea of the research.** The dissertation is devoted to the development of a nonlinear trajectory control method for a serial robot manipulator under parameter uncertainties and external disturbances. The research combines differential flatness, a geometric representation of dynamics, and robust correction of the computed torque control method. This approach aims to improve tracking accuracy when the nominal and actual models of the controlled system differ. It employs the concept of Type A differentially flat systems, whose flat outputs depend exclusively on state variables. A fully actuated serial robot manipulator with joint torque control is considered a system of this type, providing a theoretical basis for applying the generalized computed torque method to trajectory control.

The following approaches have been proposed and substantiated:

- representation of a serial manipulator with torque actuation as a Type A differentially flat system and generalization of the computed torque method;
- geometric description of the dynamics based on the kinetic energy metric and linearization of the dynamics through static state feedback;
- a nonlinear robust trajectory control law that compensates for the effects of mathematical model uncertainties and external disturbances.

The proposed theoretical results were investigated analytically and verified through computer simulations of a two-link manipulator. The classical computed torque method and the proposed robust control law were compared in terms of trajectory tracking accuracy and control torques.

**Keywords:** multi-link serial robot manipulator, nonlinear control, computed torque control, differential flatness, geometric dynamics, feedback linearization, robust control.

**The relevance of the research.** The expanding application of robot manipulators is accompanied by increasing requirements for accurate trajectory tracking, movement speed, and robustness under changing operating conditions. The dynamics of a serial manipulator are nonlinear and strongly coupled: the motion of each joint affects the other links, while inertial, Coriolis, centrifugal, and gravitational components depend on the configuration and velocity of the manipulator.

The computed torque method makes it possible to compensate for nonlinearities and establish the desired error dynamics. However, its effectiveness depends on the accuracy of the mathematical model. Uncertainties in masses and moments of inertia, unmodeled friction, external disturbances, and changes in payload compromise this compensation and deteriorate tracking performance.

Therefore, it is necessary to combine the advantages of exact linearization with a mechanism for suppressing residual uncertainties.

Differential flatness establishes the relationship between system states, control inputs, and the desired trajectory, while the geometric representation relates the dynamics to the metric of the configuration space. The combined use of these approaches makes it possible to justify the control structure and formulate conditions for improving its robustness. The development of a corresponding nonlinear control law and verification of its effectiveness constitute a relevant scientific problem.

The dissertation research was carried out within the framework of two projects funded by grants from the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan: No. AP19679327, "Machine Learning Methods for Automatic Control and Inertial Navigation of Mobile Robots," under the priority area "Information, Communication and Space Technologies" and the subpriority "Intelligent Robotic Systems"; and No. AP13068317, "Development of New Control Algorithms for a Robot Manipulator for 3D Scanning and Additive Microplasma Coating Deposition Technologies."

**Object of the research.** A multi-link serial robot manipulator with torque actuation, considered as a nonlinear control system.

**Subject of the research.** Mathematical models, feedback linearization methods, and nonlinear robust algorithms for trajectory control of a multi-link robot manipulator.

**Aim of the research.** To develop and investigate a nonlinear trajectory control method for a multi-link robot manipulator that improves the accuracy and robustness of the control system in the presence of parameter uncertainties, mathematical model inaccuracies, and external disturbances.

**Research objectives:**

1. To analyze modern manipulator control methods, including feedback linearization, computed torque control, and robust control.
2. To develop a mathematical model of serial manipulator dynamics, taking into account the coupling between links and inertial, Coriolis, centrifugal, and gravitational components.
3. To investigate the manipulator as a differentially flat system of a special type and determine its flat output.
4. To establish the relationship between the computed torque method and the general trajectory control method for differentially flat systems of the type under consideration.
5. To describe the manipulator dynamics using differential geometry through the metric of the configuration space and the covariant derivative.
6. To determine the conditions for feedback linearization and the structure of the equivalent linear system.
7. To investigate the effects of parameter uncertainties, model inaccuracies, and external disturbances on the accuracy of the classical computed torque method.
8. To synthesize a nonlinear robust trajectory control law for the linearized system.

9. To establish the stability of the closed-loop system and determine the conditions for convergence of the tracking error.

10. To conduct computer simulations of the control methods in MATLAB/Simulink and evaluate tracking accuracy, transient responses, and dynamic characteristics of the system.

11. To compare the developed robust control law with the classical computed torque method in terms of accuracy, stability, and robustness.

**Main research methods.** The research employs methods of automatic control theory, mathematical modeling, differential geometry, differential flatness theory, and feedback linearization. System stability was investigated using Lyapunov's direct method, while sensitivity to disturbances was analyzed using frequency-domain methods. Computer studies were performed in MATLAB/Simulink through numerical integration of the dynamic equations and comparison of simulation results with analytical solutions.

**Scientific statements submitted for defense:**

1. Representation of a serial robot manipulator as a differentially flat control system of a special type; the generalized computed torque method and linearization of the manipulator control system.

2. Geometric interpretation of the manipulator dynamics and linearization through static state feedback.

3. A nonlinear robust trajectory control law for a robot manipulator based on feedback linearization and compensation for mathematical model uncertainties.

**Scientific novelty of the research:**

1. The use of the representation of a multi-link serial manipulator with torque actuation as a Type A differentially flat system has been substantiated for linearizing nonlinear dynamics and developing a trajectory control system.

2. It has been established that the classical computed torque method is a particular case of the general trajectory control method for Type A differentially flat systems.

3. A nonlinear robust control law has been developed for the linearized dynamics of a multi-link manipulator, taking into account parameter uncertainties, model inaccuracies, and external disturbances.

4. Conditions for the stability of the closed-loop system and convergence of the trajectory tracking error have been obtained for the proposed control law under the adopted assumptions.

**Main scientific results of the research:**

1. The transformation of the nonlinear dynamics of a fully actuated manipulator into a system of independent double integrators under the assumption of exact knowledge of model parameters was investigated. The structure of the generalized computed torque method was substantiated, and transient responses and the sensitivity of tracking errors to additive disturbances were examined. The analytical amplitude-frequency characteristics were found to be consistent with the results of computer simulations.

2. The geometric characteristics of a two-link manipulator model, including Christoffel symbols and Gaussian curvature, were obtained. Under constant

potential energy and in the absence of control torques, free motion was interpreted as geodesic motion. Numerical verification demonstrated agreement between the classical and geometric representations: the discrepancy between trajectories during controlled motion was $1.8 \times 10^{-11}$ rad, while the discrepancy obtained during verification of the linearization was $1.0 \times 10^{-11}$ rad.

3. The effectiveness of robust correction was confirmed under inaccurate model parameters and unmodeled friction. A 3 N·m external torque pulse and an increase of 0.5 kg in the mass of the second link were investigated separately. Under the considered operating conditions, tracking errors were reduced by a factor of 37–50 compared with the classical computed torque method. Under the pulse disturbance, the maximum joint errors decreased from 0.160 and 0.525 rad to 0.0043 and 0.0142 rad, respectively, while maintaining comparable peak actuator torques.

When the masses and moments of inertia in the model were varied within ±40%, the maximum tracking error norm of the robust control system ranged from 0.0018 to 0.0058 rad, whereas the corresponding values for classical control ranged from 0.08 to 0.29 rad. The effects of robust correction gain and boundary layer width on tracking accuracy and the smoothness of control torques were demonstrated. These results characterize the operating conditions considered in the computer simulations.

**Publications.** A total of **8** scientific works have been published on the dissertation topic, including **2** articles in international peer-reviewed scientific journals indexed in Scopus and Web of Science Core Collection: Sensors (2025, Q1 in Scopus, Q2 in WoS, JCR 2025) and Siberian Electronic Mathematical Reports (2025, Q3 in WoS, JCR 2024); **3** articles in scientific journals recommended by the authorized body in the field of science and higher education of the Republic of Kazakhstan; and **3** publications in the proceedings of international scientific conferences.

**Presentation and discussion of research results.** The main results of the dissertation research were presented and discussed at three international conferences:

1. VIII International Scientific and Technical Conference "Creativity of Youth – for the Innovative Development of Kazakhstan," April 7–8, 2022, Ust-Kamenogorsk, Kazakhstan;
2. 18th International Symposium on Applied Informatics and Related Areas (AIS 2023), November 16, 2023, Székesfehérvár, Hungary;
3. X International Scientific and Technical Conference "Creativity of Youth – for the Innovative Development of Kazakhstan," dedicated to the 125th anniversary of Kanysh Satpayev, April 11–12, 2024, Ust-Kamenogorsk, Kazakhstan.

**Practical significance.** The developed models and algorithms can be used in the design, analysis, and computer-based investigation of trajectory control systems for serial robot manipulators. The research results have been implemented in the educational process at D. Serikbayev East Kazakhstan Technical University within the educational program 6B07101 "Automation and Control" and are used in teaching the course "Nonlinear Automatic Control Systems" (Certificate of Implementation of Research Results in the Educational Process dated February 28, 2024).

**Personal contribution of the author.** The author's personal contribution includes defining the research problem, formulating the aim and objectives of the study, developing mathematical models and control methods, performing analytical calculations and computer experiments, analyzing and interpreting the results, preparing scientific publications, and presenting and implementing the research findings.

**Structure and scope of the dissertation.** The dissertation consists of an introduction, four chapters, a conclusion, a list of references, and Appendix A. The dissertation comprises 151 pages, including 81 figures and 7 tables. The list of references contains 92 sources.

The chapters sequentially address modern methods for controlling multi-link manipulators, differential flatness and the generalized computed torque method, the geometric interpretation of dynamics and its linearization, as well as nonlinear robust trajectory control and the results of computer simulations.